CHAPTER 1: Real Numbers

- **Fundamentals:**
  1. A non-negative integer 'p' is said to be divisible by an integer 'q' if there exists an integer 'd' such that:
  $$p = qd.$$
  2. ± 1 divides every non-zero integer.
  3. 0 does not divide any integer.
- **Fundamental Theorem of Arithmetic:**
  Every composite number can be expressed as a product of primes, and this factorisation is unique except for the order in which the prime factors occur.

  Important Theorems:
  1. Let p be a prime number and a be a positive integer. If p divides a^2, then p divides a.
  2. Consider two positive integers a and b, then LCM × HCF = $a \times b$.

  HCF and LCM:
  1. HCF (a, b): Product of the smallest power of each common prime factor in the numbers.
  2. LCM (a, b): Product of the greatest power of each prime factor, involved in the numbers.
  3. HCF (a, b) × LCM (a, b) = $a \times b$, for any two positive integers a and b.

CHAPTER 2: Polynomials

- **Fundamentals:**
  An algebraic equation of the form,
  $$p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots\dots\dots + a_{n-1}x + a_n$$
  is called polynomial, provided it has no negative exponent for any variable.
  where, $a_0, a_1, a_2, \dots\dots a_{n-1}, a_n$ are constants (real numbers); $a_0 \neq 0$.
- **Degree of polynomial:** n is called the degree (highest power of variable x). If $n = 1$ then polynomial is called linear polynomial.
  General form: $ax + b$ where, $a \neq 0$
  If $n = 2$ then polynomial is called quadratic polynomial.
  General form: $ax^2 + bx + c$ where, $a \neq 0$
  If $n = 3$ then polynomial is called cubic polynomial.
  General form: $ax^3 + bx^2 + cx + d$ where, $a \neq 0$
- **Zeroes of polynomial:** For polynomial $p(x)$, the value of x for which $p(x) = 0$, is called zero(es) of polynomial.
  Linear polynomial can have atmost 1 root (zero).
  Quadratic polynomial can have atmost 2 roots (zeroes).
  Cubic polynomial can have atmost 3 roots.

- **Relationship between zeroes and coefficient of polynomial:**
  - Zero of linear polynomial $ax + b$ is given by $x = \frac{-b}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x)}$.
  - If α and β are zeroes of the quadratic polynomial $ax^2 + bx + c$, then

    Sum of zeroes, $\alpha + \beta = \frac{-b}{a} = -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$.

    Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$.
  - If α, β and γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

    Sum of zeroes, $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$.

    Product of roots taken two at a time

    $$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^3)}.$$

    Product of zeroes, $\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x^3)}$.

CHAPTER 3: Pair of Linear Equations in Two Variables

- **Fundamentals:**
  1. The general form of linear equation in one variable is: $ax + b = 0, (a \neq 0)$.
  2. The general form of linear equation in two variables is: $ax + by + c = 0$.
     where, a, b, c are real coefficients; ($a^2 + b^2 \neq 0$ i.e., a and b are not both zero).
  3. Every linear equation gives a straight line graph. Every point lying on this line is a solution of linear equation.
  4. Two linear equations which are in the same two variables x and y simultaneously are called pair of linear equations in two variables.
  5. The general form of pair of linear equation in two variables is:
     $$a_1x + b_1y + c_1 = 0$$
     and
     $$a_2x + b_2y + c_2 = 0$$
     where, $a_1, b_1, c_1, a_2, b_2, c_2$ are all real coefficients.
  6. A pair of values of x and y satisfying each one of the equations is called solution of the system.
- **Graphical and Algebraic Interpretation:**

Pair of linear equations	Algebraic condition	Graphical interpretation	Algebraic interpretation	consistency
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Dependent Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

Note: A pair of linear equations having atleast one solution is called consistent otherwise inconsistent

- **Methods of Solving Pair of Linear Equations:**
  - (A) Graphical method
  - (B) Algebraic methods
    - (i) Substitution method
    - (ii) Elimination method

Graphical Method:

Step 1: Plot both the linear equations on the same graph.

Step 2: Find the intersecting point on graph, if the lines are intersecting.

Step 3: Intersecting point is the required solution.

Step 4: In step 2, if the lines are coincident, then there are infinitely many solutions—each point on the line being a solution.

Step 5: In step 2, if the lines are parallel, then the pair of equations has no solution.

Substitution Method:

Step 1: From one equation, find the value of one variable in terms of other variable.

Step 2: Substitute the value of variable obtained in step 1 in the other equation, you will get the equation in one variable.

Step 3: Solve the equation in one variable and find the value of variable.

Step 4: Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.

Step 5: Solve this equation in one variable and find the value of this variable.

Elimination Method:

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2: Then add or subtract one equation from other so that one variable gets eliminated and resultant equation will become an equation in one variable.

Step 3: Solve the equation in one variable and find the value of the variable.

Step 4: Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.

Step 5: Solve this equation in one variable and find the value of this variable now.

Equations Reducible to a pair of linear equations in two variable: There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

CHAPTER 4 : Quadratic Equations

- **Fundamentals:**
  1. General form of quadratic equation is: $ax^2 + bx + c = 0$, where $a \neq 0$.
  2. a is co-efficient of x^2, b is co-efficient of x, c is called constant term.
  3. Equation of the form $ax^2 + c = 0$ is called pure quadratic equation.
  4. The value of variable satisfying equation is called root of that equation.
  5. Quadratic equation has atmost two roots.
- **Methods of Solving Quadratic Equations:**
  - (a) Factorisation method (splitting the middle term)
  - (c) Quadratic formula method (Sridharacharya formula)

Factorisation Method:

Step 1: Resolve the equation in factor using splitting the middle term method, i.e,: $ax^2 + bx + c = (Ax + B)(Cx + D)$

Step 2: Put both factors equal to zero,

$Ax + B = 0$ and $Cx + D = 0$

Therefore, $x = -\dfrac{B}{A}$ and $x = -\dfrac{D}{C}$ are two roots.

- **Quadratic Formula Method:**
Direct formula to calculate the roots is given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof: Use steps given in completing the square method.
Discriminant: It is denoted by 'D' and given by:

$$D = b^2 - 4ac$$

- **Nature of Roots of Quadratic Equation:**
First of all find D.
  - If $D > 0$, roots are real and unequal.
  - If $D = 0$, roots are real and equal.
  - If $D < 0$, roots are imaginary.

CHAPTER 5: Arithmetic Progressions

- **Sequence:** An arrangement of numbers which has a pattern, which can suggest the successor of every number in the arrangement.
Examples of Arithmetic Progressions:
- **Arithmetic Progression (AP):** It is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
(i) $3, 5, 7, 9, 11$..........
(ii) $-8, -5, -2, 1, 4, 7$........
(iii) $6, 1, -4, -9, -14$.........
(iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ .... Yes it is AP, because it can be written as below
$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$ ....
(v) $3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, 3 + 4\sqrt{2}, 3 + 5\sqrt{2}$ ....
This fixed number is called common difference, denoted by 'd'. It can be positive or negative.
So, general form of an AP is given by:
$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots, a + (n-1)d.$
where, a = first term, d = common difference and $a + (n-1)d = n^{th}$ term.
If t_1, t_2, t_3 .... be the I, II, III...... terms of an AP.
then, $t_2 - t_1 = d$
$t_3 - t_2 = d$
$t_4 - t_3 = d$ and so on......................
Hence, It can be written:
$t_2 - t_1 = t_3 - t_2$
$2t_2 = t_1 + t_3$
Conclusion: If three numbers a, b and c are in AP then:
$2b = a + c.$
- **Important Formulae:**
nth term of an AP $T_n = a + (n-1)d$
Sum of the n terms of an AP is: $S_n = \frac{n}{2}[2a + (n-1)d]$
Also: $S_n = \frac{n}{2}(a + l)$
where, l = last term, i.e., $l = a + (n-1)d$.

1. n^{th} term from the end of an AP: $(l-(n-1)d)$
2. $t_n = (S_n - S_{n-1})$

- **Tips:**

1. To an AP if we (i) add (ii) subtract (iii) multiply or (iv) divide each term by the same number, the resulting sequence would always be an AP.
2. Whenever you be asked to take three numbers which are in AP, always take:
   $a-d, a, a+d$.
3. Whenever you be asked to take four numbers which are in AP, always take:
   $(a-3d), (a-d), (a+d), (a+3d)$
4. Whenever you be asked to take five numbers which are in AP, always take:
   $(a-2d), (a-d), (a), (a+d), (a+2d)$

- **Proof of "Sum of n terms of an AP"**

We know that general form of an AP is given by:

$a, a+d, a+2d, a+3d, a+4d, \ldots\ldots\ldots\ldots, a+(n-1)d.$

$\Rightarrow \; S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots\ldots\ldots + [a+(n-2)d] + [a+(n-1)d]$ (i)

Now write the above equation in reverse order:

$\Rightarrow \; S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots\ldots\ldots\ldots\ldots\ldots + (a+3d) + (a+2d) + (a+d) + a$ (ii)

Adding the corresponding terms of eq (i) & (ii), we get

$\Rightarrow \; 2S_n = [2a+(n-1)d] + [2a+(n-1)d] + [2a+(n-1)d] \ldots\ldots\ldots\ldots\ldots\ldots [2a+(n-1)d]$

$\Rightarrow \; 2S_n = n[2a+(n-1)d]$

$\Rightarrow \; S_n = \frac{n}{2}[2a+(n-1)d]$

Also, it can be written as: $S_n = \frac{n}{2}[a+a+(n-1)d]$

$\Rightarrow \; S_n = \frac{n}{2}[a+l]$ [where, last term, $l = a+(n-1)d$] **Hence Proved.**

- **Some important key points:**

1. We know, $a_n = a+(n-1)d$
   $a_n = a + nd - d$
   $a_n = (a-d) + nd$

i.e., Linear equation denotes general term where,
(i) co-efficient of n is common difference 'd'
(ii) constant term is $(a-d)$.

2. We know, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_n = na + \frac{n}{2}(n-1)d$$

$$S_n = na + n^2\frac{d}{2} - n\frac{d}{2}$$

$$S_n = n\left(a - \frac{d}{2}\right) + n^2\frac{d}{2}$$

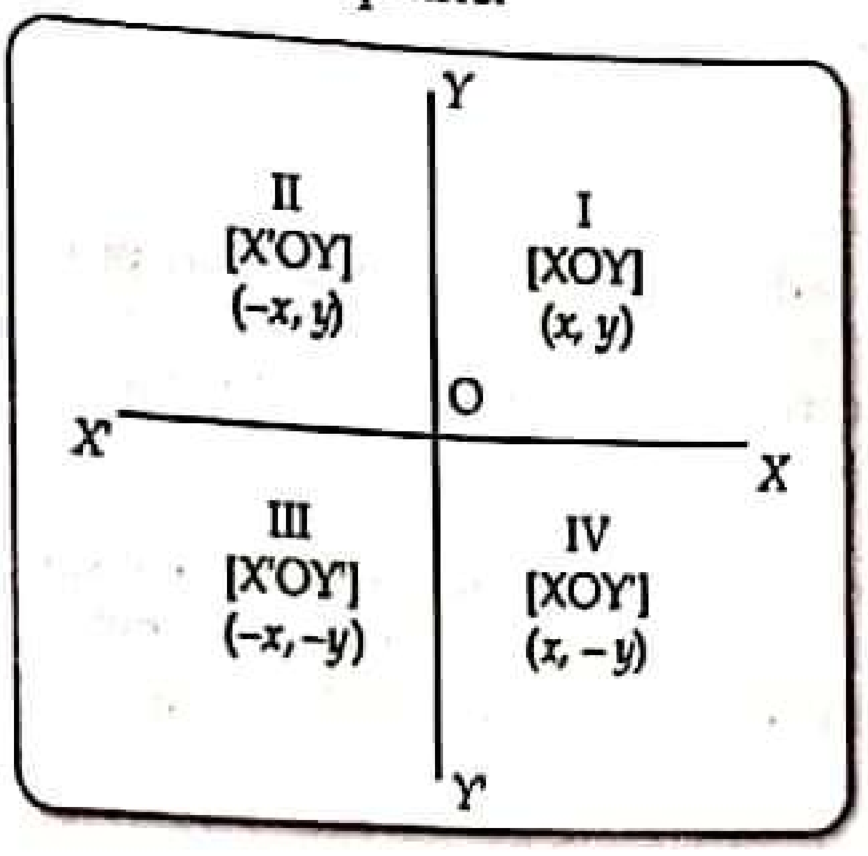
i.e., Quadratic equation denotes sum to n terms where,

(i) Co-efficient of n^2 is $\frac{d}{2}$

(ii) Co-efficient of n is $\left[a - \frac{d}{2}\right]$

CHAPTER 6: Coordinate Geometry

- **Fundamentals:**
  - (i) Distance of any point from the y-axis is called x co-ordinate or abscissa.
  - (ii) Distance of any point from the x-axis is called y co-ordinate or ordinate.
  - (iii) Origin: (0, 0)
  - (iv) Point on x-axis: $(x, 0)$.
  - (v) Point on y-axis: $(0, y)$
  - (vi) There are four quadrants in a co-ordinate plane:



- **Distance Formula:**

Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Corollary: Distance of point $A(x, y)$ from origin is $\sqrt{x^2 + y^2}$

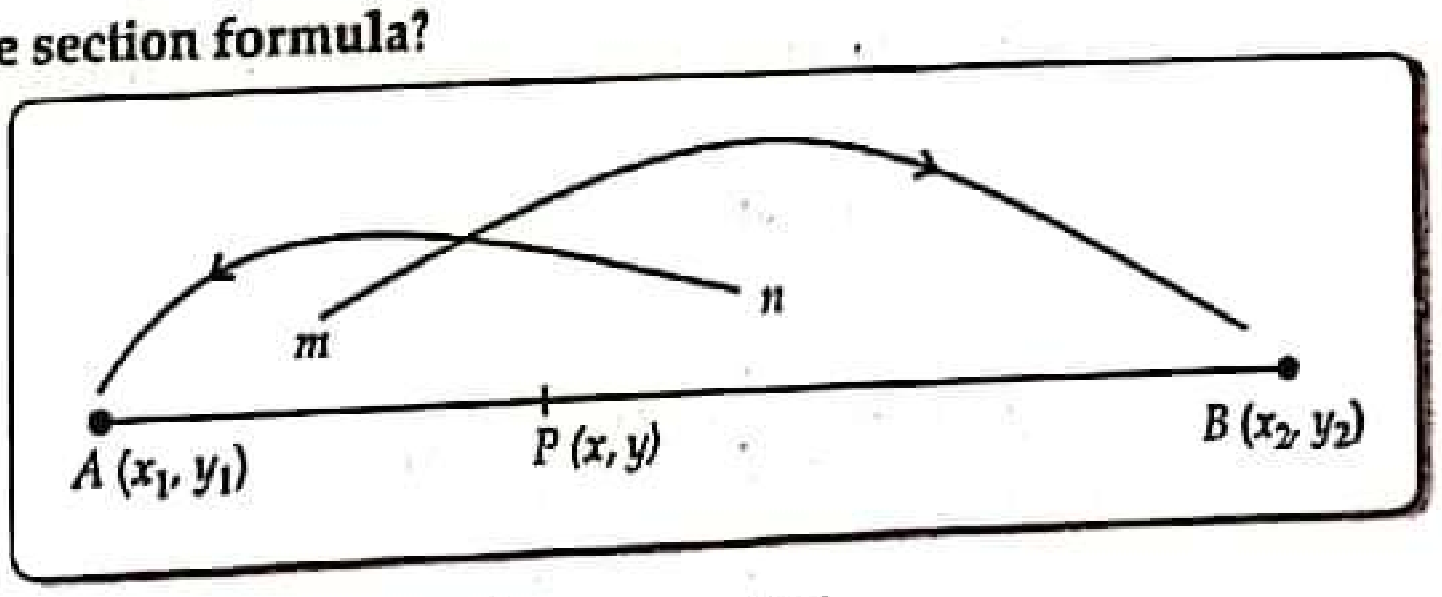
- **Tips:** Co-ordinates will form:
  1. Rhombus, if all the four sides are equal.
  2. Square, if all the four sides and diagonals are equal.
  3. Parallelogram, if opposite sides are equal.
  4. Rectangle, if opposite sides and diagonals are equal.
  5. Right triangle, if it follows Pythagoras theorem.
  6. Collinearity condition. [A, B, C are collinear if $AB + BC = AC$]

- **Section Formula:**

Co-ordinates of the point $P(x, y)$, dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by:

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

- **How to remember the section formula?**



- **Corollary:** If $P(x, y)$ is the mid-point, therefore $m : n = 1 : 1$

$$x = \frac{x_2 + x_1}{2}, y = \frac{y_2 + y_1}{2}$$

- **Tips:** If the ratio in which P divides AB is not given then we take assumed ratio as $k : 1$.

- **Fundamentals:**

Similar figures: Two figures of same shape are said to be similar. if:
1. Their corresponding angles are equal.
2. Their corresponding sides are proportional.

Examples:
1. All circles
2. All squares
3. All equilateral triangles
4. All congruent triangles.

- **Criterion for similarity of two triangles**

(i) **SSS Similarity:** If the corresponding sides of two triangles are proportional, then triangles are similar.
(ii) **AAA Similarity:** If the corresponding angles of two triangles are equal, then triangles are similar.
(iii) **SAS Similarity:** If the pair of corresponding sides of two triangles are proportional and the included angle is equal, then triangles are similar.

- **Statements of theorems**

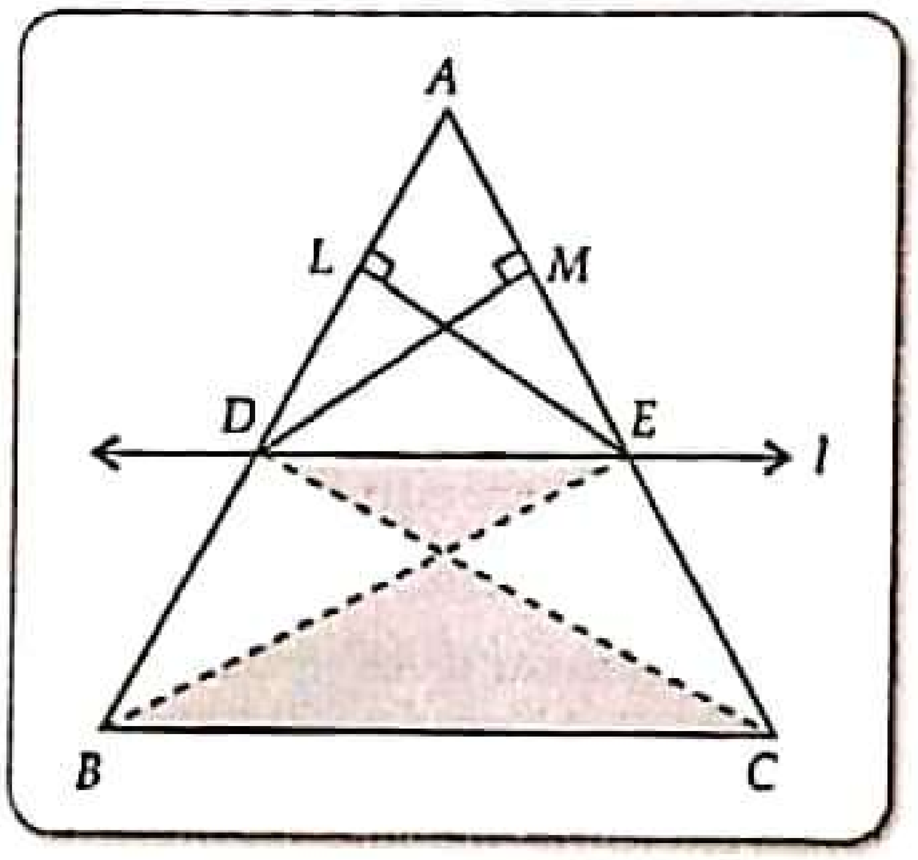
1. **Basic proportionality theorem:** If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio.
2. **Converse of basic proportionality theorem:** If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

Proof of Theorems:

1. **Basic proportionality theorem (Thales theorem):**

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



To Prove: $$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Join BE and CD. Draw $EL \perp AB$ and $DM \perp AC$.

Proof: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have

$$\text{area}(\Delta BDE) = \text{area}(\Delta CDE) \qquad \text{(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \qquad \text{(ii)}$$

Put value from (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \qquad \text{(iii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} = \frac{AE}{EC} \quad \text{(iv)}$$

On comparing equations (iii) and (iv), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

Corollary:

(i) $\frac{AB}{DB}=\frac{AC}{EC}$

(ii) $\frac{DB}{AD}=\frac{EC}{AE}$

(iii) $\frac{AB}{AD}=\frac{AC}{AE}$

(iv) $\frac{DB}{AB}=\frac{EC}{AC}$

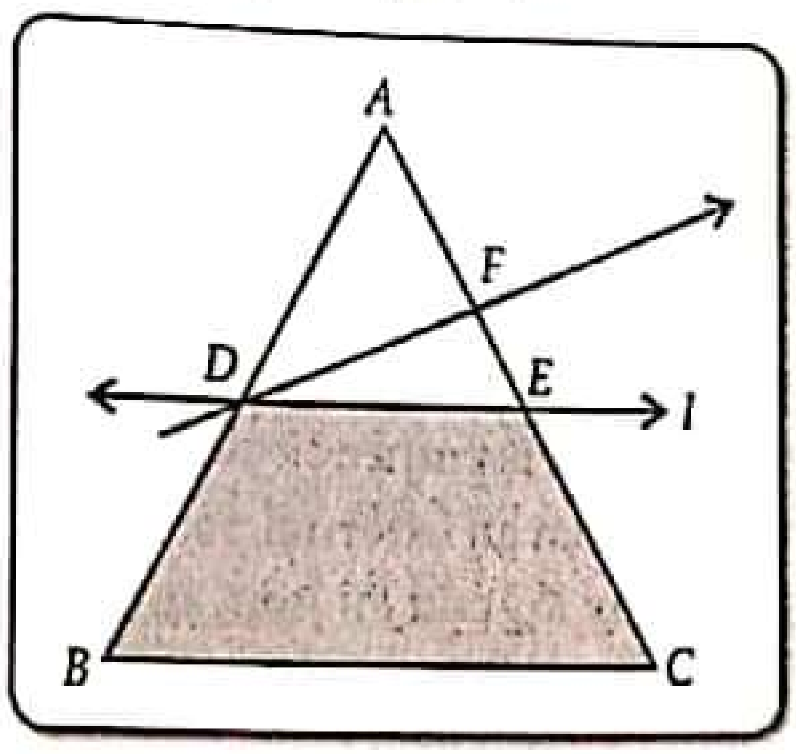
(v) $\frac{AD}{AB}=\frac{AE}{AC}$

2. **Converse of Basic Proportionality:**

Statement: If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

Given: A ΔABC and line 'l' intersecting the sides AB at D and AC at E such that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



To Prove: $l \parallel BC$.

Proof: Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{(Basic proportionality theorem) (i)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(Given) (ii)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

$$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC}$$

$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$

$\Rightarrow \quad FC = EC$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$. Hence, there one and only line, $DE \parallel BC$, i.e.,

$$\boxed{l \parallel BC}$$

Hence Proved.

CHAPTER 8: Circles

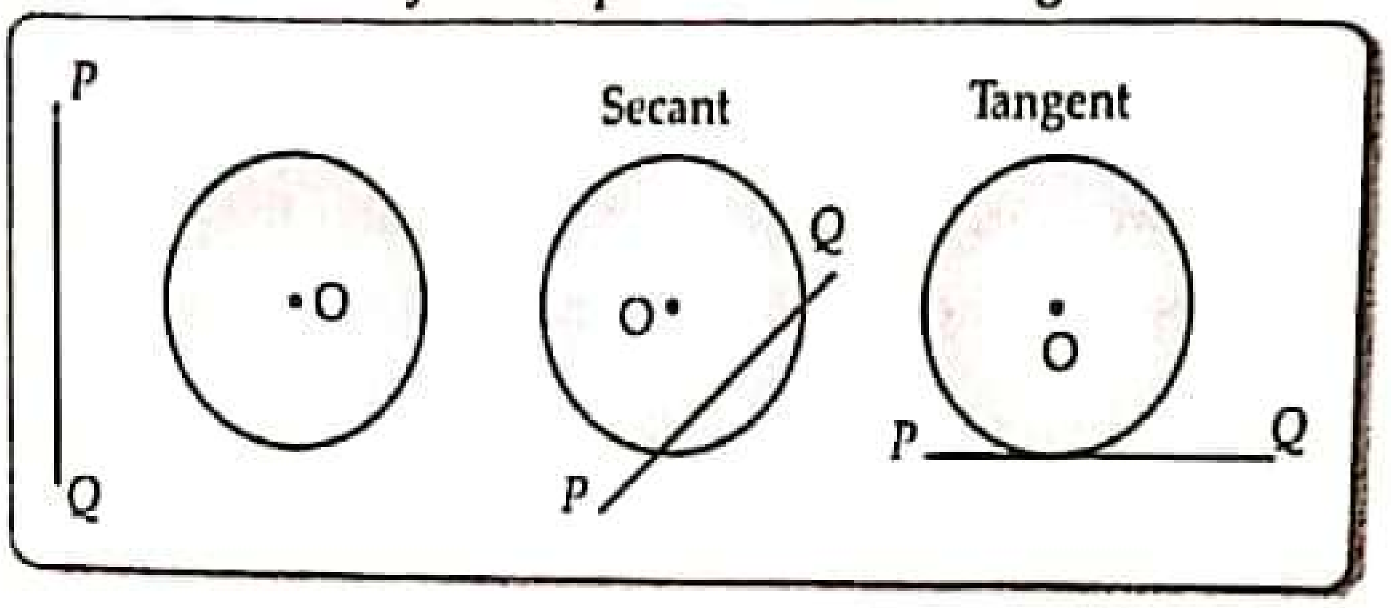
- **Fundamentals:**

  Consider a circle $C(O, r)$ and a line PQ. There can be three possibilities given below:

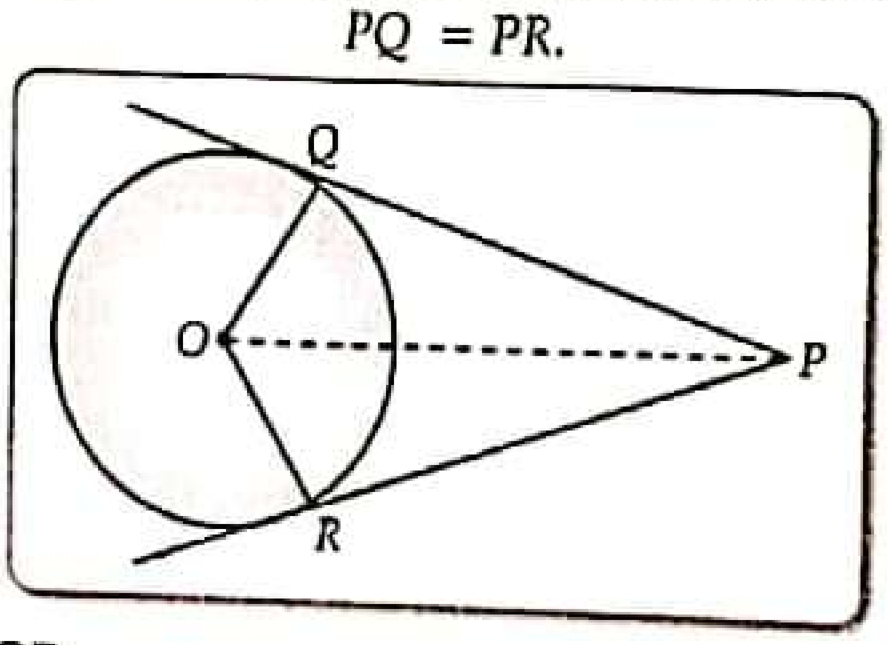
  (i) Non intersecting line w.r.t. circle

  (ii) A line intersects circle in two distinct points, this line is called a **Secant.**

  (iii) A line which intersects circle exactly at one point is called a **Tangent.**



From a point P inside a circle, the number of tangents drawn to the circle = 0.
From a point P on a circle, the number of tangents drawn to the circle = 1.
From a point P outside the circle, the number of tangents drawn to the circle = 2.
The distance between two parallel tangents drawn is equal to the diameter of the circle.
Theorem 1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.
Theorem 2: The lengths of two tangents from an external point to a circle are equal.
Given: A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.
To prove: $PQ = PR$.



Construction: Join OQ, OR and OP.

Proof: In ΔOQP and ΔORP

$OQ = OR$ (radii of the same circle)

$OP = OP$ (Common)

$\angle Q = \angle R$ = each 90° (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

Hence, $\Delta OQP \cong \Delta ORP$ (By RHS criterion)

$\therefore$ $PQ = PR$ (By c.p.c.t.)

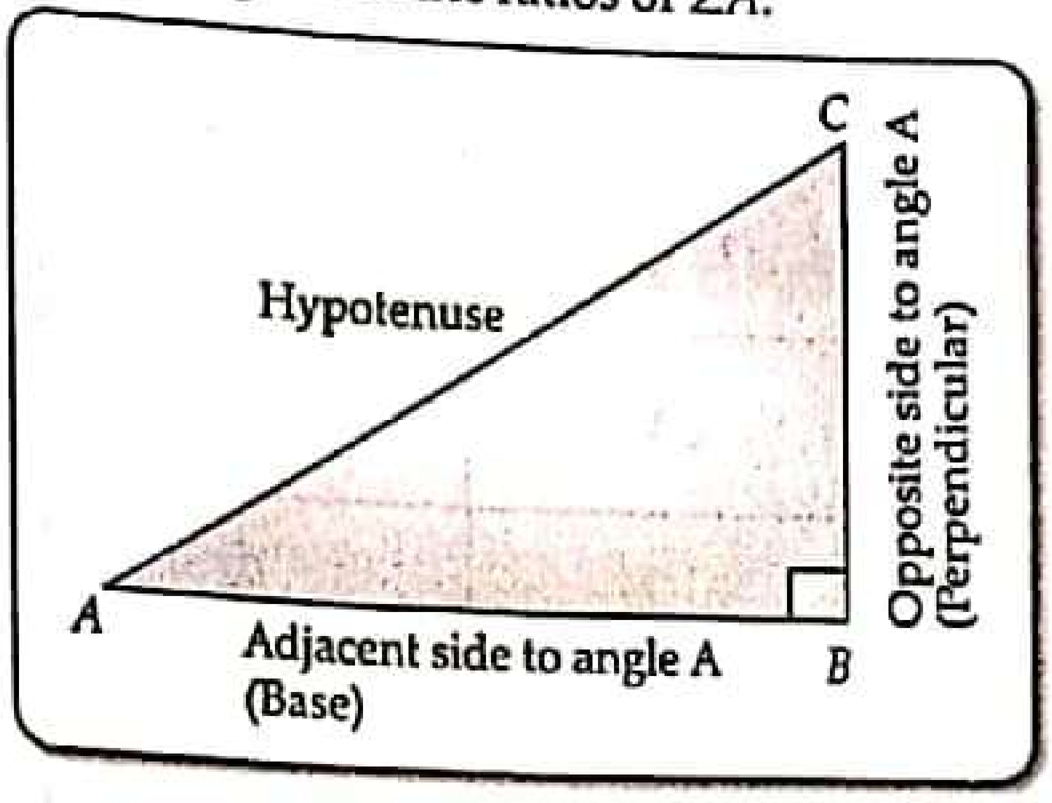
Hence Proved.

Theorem 3: Tangents are equally inclined on the line segment joining external point and centre.

Theorem 4: Tangent subtend equal angle at the centre.

CHAPTER 9: Introduction to Trigonometry and Trigonometric Identities

- Trigonometry is the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.
- **Trigonometric Ratios:** The values of the ratios of the sides of any right triangle with respect to any angle (other than 90°) are called trigonometric ratios of that angle. For example: In right $\angle ABC$, the ratios of the sides of the triangle with respect to $\angle A$ are called trigonometric ratios of $\angle A$.



There are six different trigonometric ratios as follows:

1. Sine A $= \dfrac{\text{Opposite side to angle } A}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$
2. Cosine A $= \dfrac{\text{Adjacent side to angle } A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{\text{Base}}{\text{Hypotenuse}}$
3. Tangent A $= \dfrac{\text{Opposite side to angle } A}{\text{Adjacent side to angle } A} = \dfrac{BC}{AB} = \dfrac{\text{Perpendicular}}{\text{Base}}$
4. Cosecant A $= \dfrac{\text{Hypotenuse}}{\text{Opposite side to angle } A} = \dfrac{AC}{BC} = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$
5. Secant A $= \dfrac{\text{Hypotenuse}}{\text{Adjacent side to angle } A} = \dfrac{AC}{AB} = \dfrac{\text{Hypotenuse}}{\text{Base}}$
6. Cotangent A $= \dfrac{\text{Adjacent side to angle } A}{\text{Opposite side to angle } A} = \dfrac{AB}{BC} = \dfrac{\text{Base}}{\text{Perpendicular}}$

Tips:

1. sin A is written for sine A.
2. cos A is written for cosine A.
3. tan A is written for tangent A.
4. cosec A is written for cosecant A.
5. sec A is written for secant A.
6. cot A is written for cotangent A.

Relation between Trigonometric ratios:

$\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ OR $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ OR $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ OR $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ OR $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

- **Trigonometric ratios of some specific angles:**

In this part, we will put values of angles as 0°, 30°, 45°, 60° and 90°, hence we will find ratios.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

- **How to remember trigonometric ratios of some specific angles?**

1. First of all learn only sin row. If you can't learn then follow the step to find $\sin\theta$ row:

θ	0°	30°	45°	60°	90°
$\sin\theta$	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{4}}$	$\frac{\sqrt{2}}{\sqrt{4}}$	$\frac{\sqrt{3}}{\sqrt{4}}$	$\frac{\sqrt{4}}{\sqrt{4}}$
	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1

2. For $\cos\theta$ row, write all the values of $\sin\theta$ row in reverse order, i.e., from right to left.

- **Fundamental Trigonometric Identities:**

There are three fundamental identities which can be written in six different ways.

1. $\sin^2\theta + \cos^2\theta = 1$

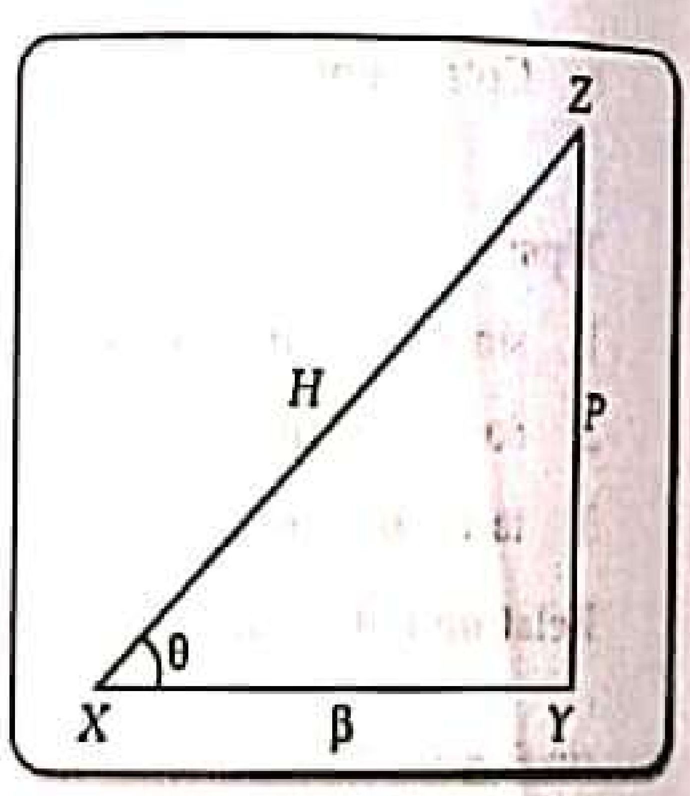
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

2. $\sec^2\theta - \tan^2\theta = 1$

$$\tan^2\theta = \sec^2\theta - 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$

3. $\text{cosec}^2\theta - \cot^2\theta = 1$

$$\cot^2\theta = \text{cosec}^2\theta - 1$$
$$\text{cosec}^2\theta = 1 + \cot^2\theta$$



Proof of first identity: $\sin^2\theta + \cos^2\theta = 1$

Let P denotes Perpendicular, B denotes Base, and H denotes Hypotenuse.

In ΔXYZ, $\angle Y = 90°$, $\sin\theta = \frac{P}{H}$ and $\cos\theta = \frac{B}{H}$

$$\text{LHS} = \sin^2\theta + \cos^2\theta$$ (putting values of $\sin\theta$ and $\cos\theta$)

$$= \frac{P^2}{H^2} + \frac{B^2}{H^2}$$

$$\Rightarrow \quad = \frac{P^2 + B^2}{H^2}$$

$$= \frac{H^2}{H^2}$$

Therefore,

$$= 1 = \text{R.H.S.}$$

Hence Proved.

Similarly, other two identities can be proved.

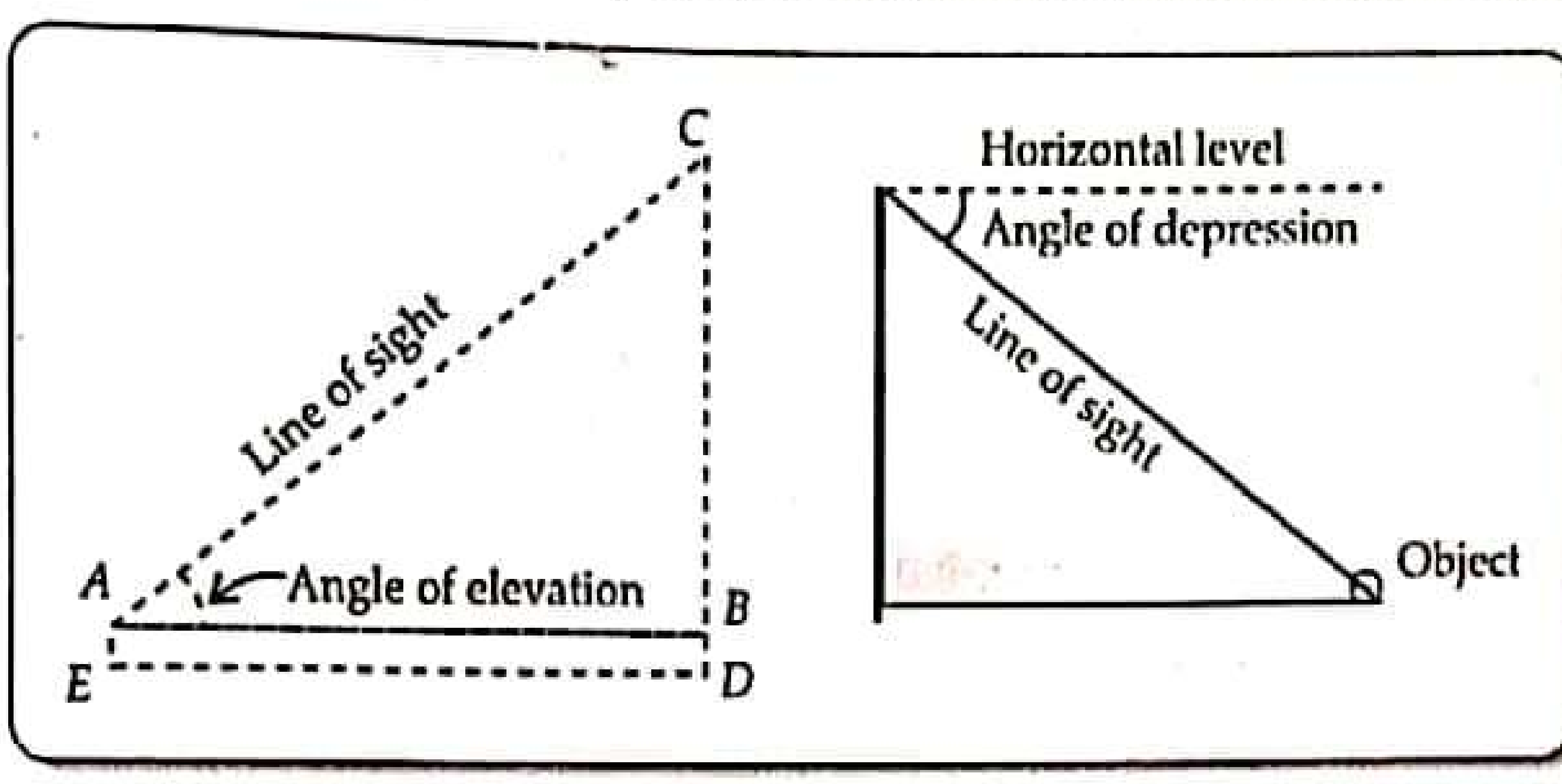
CHAPTER 10: Heights and Distances

- **Angle of Elevation:**

  The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

  The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal.

  When the point being viewed is above the horizontal level, *i.e.*, the case when we raise our head to look at the object is also shown in diagram that we have to assume a horizontal level at our eyes.



- **Angle of Depression:**

  The **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal.

  When the point is below the horizontal level, i.e.,the case when we lower our head to look at the point being viewed.

CHAPTER 11: Areas Related to Circles

- **Fundamentals:**

1. Circle is defined as the set of all those points which are at a constant distance from a fixed point. The fixed point is called **centre.**
2. The constant distance is called **radius.**
3. The longest chord passing through centre and whose end point lies on circle is called **diameter.**
4. Circles with same centre are called **concentric circles.**
5. Perimeter of circle is called **circumference.**
6. π is defined as the ratio of circumference and diameter of circle.

i.e.,

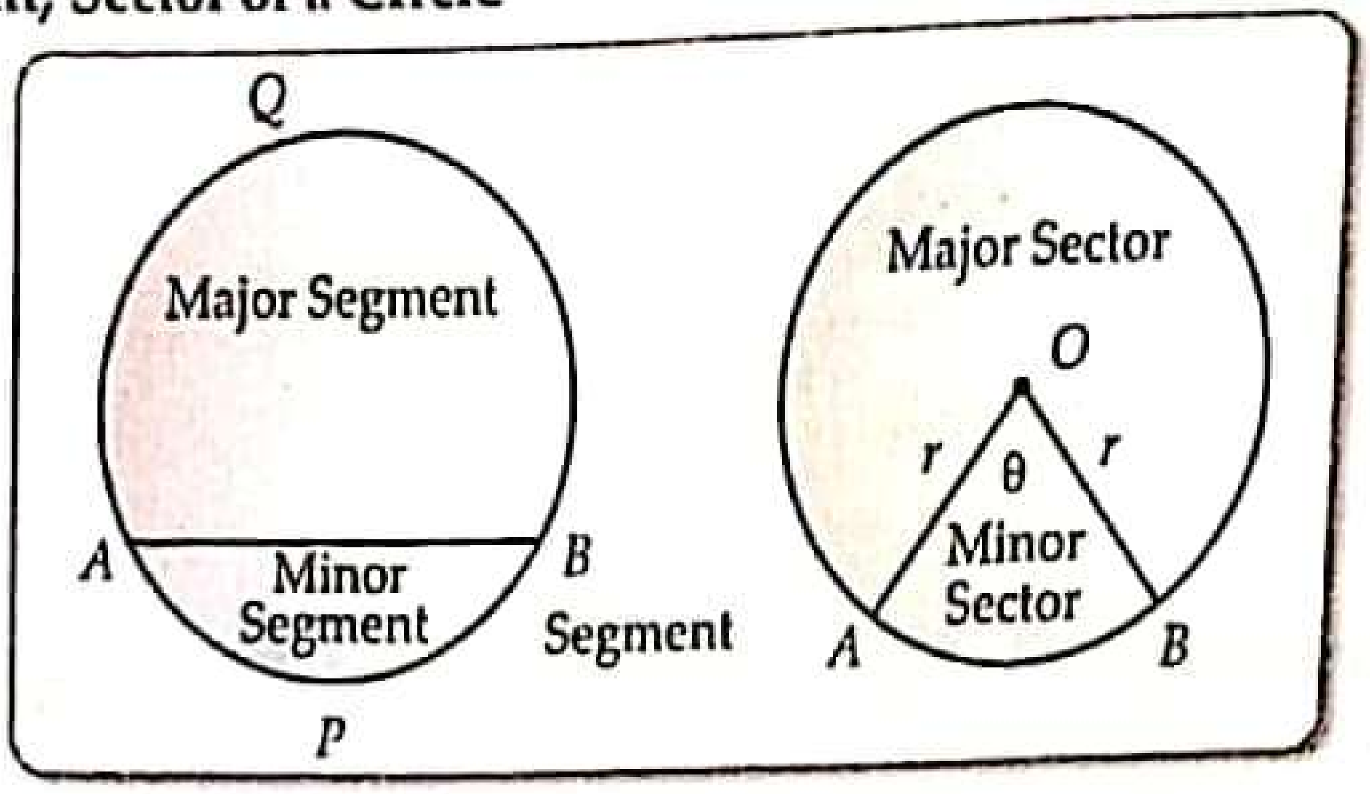
$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$$\therefore \quad \text{Circumference} = \pi \times \text{diameter}$$

$\Rightarrow$ Circumference $= 2\pi r$ [where, r is the radius of circle]

and Area of the Circle $= \pi r^2$

7. **Arc, Chord, Segment, Sector of a Circle**



(i) **Arc:** Any portion of circumference. i.e., APB is minor arc while AQB is major arc.

(ii) **Chord:** The line joining any two points on the circle. i.e., AB.

(iii) **Segment:** In figure, chord AB divides the circle in two segments i.e., $APBA$ (minor segment) and $AQBA$ (major segment).

(iv) **Sector:** The region bounded by the two radii AO and BO and arc AB is called sector of the circle.

8. **Length of Arc:** When sector angle $\angle AOB = \theta$. (where, θ is called central angle)

We know that length of arc when sector angle ($\angle AOB = 360°$) is $2\pi r$

$$\text{Length of arc when sector angle } (\angle AOB = 1°) \text{ is } = \frac{2\pi r}{360°}$$

$$\text{Length of arc when sector angle } (\angle AOB = \theta) = \frac{2\pi r \times \theta}{360°}$$

$$\text{Length of arc } AB = 2\pi r \times \frac{\theta}{360°}$$

9. **Area of Sector:** When sector angle $\angle AOB = \theta$

We know that area of circle when sector angle ($\angle AOB = 360°$) is πr^2

Area of arc when sector angle ($\angle AOB = 1°$) is $\frac{\pi r^2}{360°}$

Area of arc when sector angle ($\angle AOB = \theta$) is $\frac{\pi r^2}{360°} \times \theta$

$$\therefore \quad \text{Area of Sector} = \pi r^2 \times \frac{\theta}{360°}$$

$$\text{Area of Sector} = \frac{1}{2} \times l \times r$$

l = length of arc

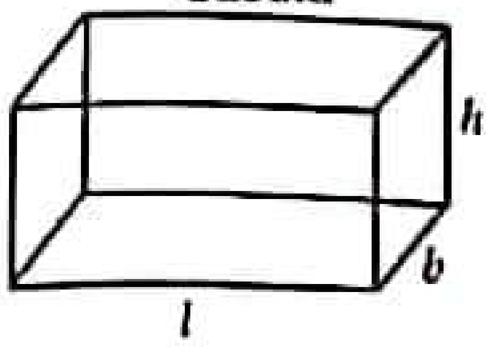
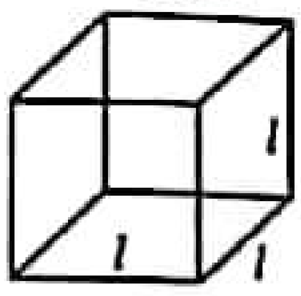
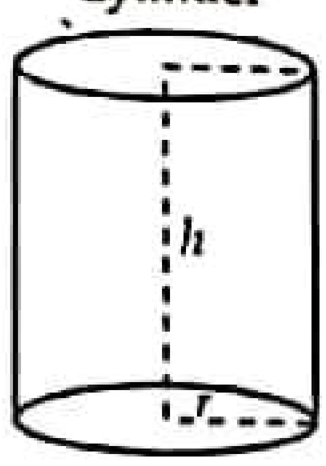
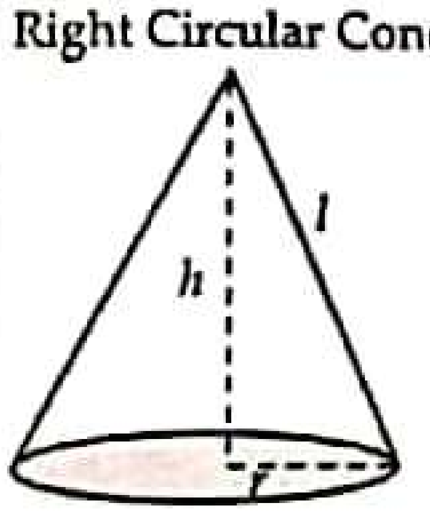
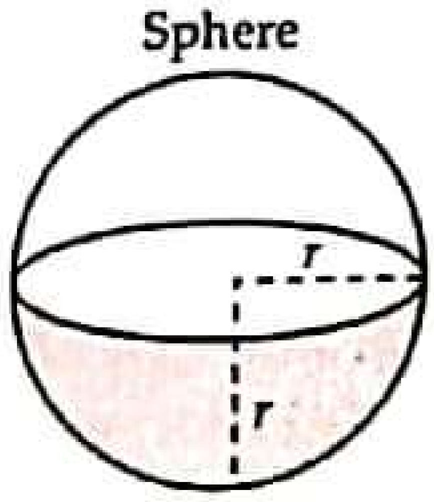
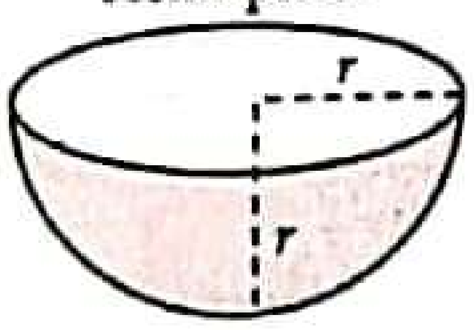
r = radius

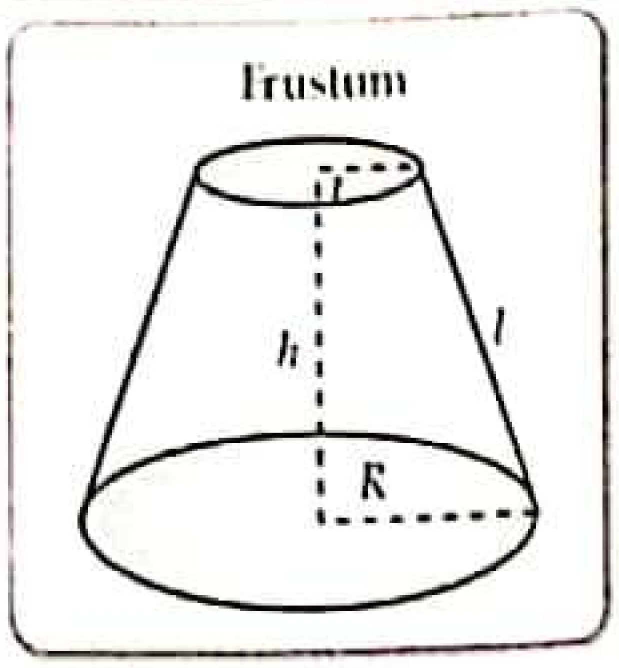
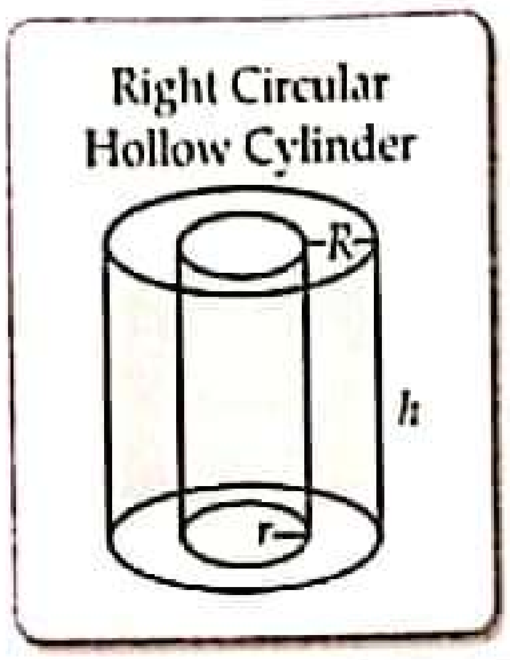
10. **Perimeter of Segment (shaded) of a Circle:** AB + arc(APB)

$$\text{Perimeter of segment} = \frac{2\pi r\theta}{360°} + 2r \sin \frac{\pi\theta}{360°}$$

CHAPTER 12: Surface Areas and Volumes

Fundamentals:

S. No.	Shape	CSA	TSA	Volume	Nomenclature
1.	Cuboid 	$2(bh + hl)$	$2(lb+bh+hl)$	lbh	l = length b = breadth h = height
2.	Cube 	$4l^2$	$6l^2$	l^3	l = length or side
3.	Right Circular Cylinder 	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height
4.	Right Circular Cone 	πrl	$\pi r(r + l)$	$\frac{1}{3}(\pi r^2 h)$	r = radius of base h = height l = slant height $l = \sqrt{r^2 + h^2}$
5.	Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}(\pi r^3)$	r = radius
6.	Hemisphere 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}(\pi r^3)$	r = radius

7.		$\pi l(r + R)$	$\pi l(r + R) + \pi(r^2 + R^2)$	$\frac{1}{3}\pi h(r^2 + R^2 + rR)$	r = radius of smaller base R = radius of larger base h = height l = slant height $l = \sqrt{h^2 + (R - r)^2}$
8.		$2\pi h(r + R)$	$2\pi(r + R)(h + R - r)$	$\pi h(R^2 - r^2)$	r = inner radius R = outer radius h = height

- **Tips:**

$$\text{Area} \times \text{Rate} = \text{Cost}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$1\ m^3 = 1000\ L$$

$$1\ m^3 = 1\ kL$$

$$1\ L = 1000\ cm^3$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$1\ km = 1000\ m = 10^5\ cm$$

$$1\ km^2 = 10^6\ m^2$$

$$1\ m = 100\ cm$$

$$1\ m^2 = 10000\ cm^2$$

$$1\ km/hr = \frac{5}{18}\ m/s$$

$$1\ km/hr = \frac{50}{3}\ m/min$$

$$\text{Shape of river} = \text{Cuboid}$$

$$1\ \text{acre} \approx 4047\ m^2$$

$$1\ \text{hectare} = 10000\ m^2$$

CHAPTER 13: Statistics

- **Fundamentals:**

1. The word statistics is used in both singular as well as plural.
2. In singular, it means "science of collection, presentation, analysis and interpretation of numerical data".

3. In plural, it means "numerical facts collected with definite purpose".
4. The number of times an observation occurs in the given data is called the frequency.
5. Frequency distribution is of two types:
   (i) Discrete frequency distribution.
   (ii) Continuous or grouped frequency distribution.
6. Class mark = (Lower limit + Upper Limit)/2.
7. The commonly used measures of central tendency are as follows:
   Arithmetic mean (MEAN), Geometric mean, Harmonic mean, Median and Mode.

(i) Relation between mean, median and mode:

$$3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$$

(ii) Mean of Grouped Data : If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$, are observations with respective frequencies $f_1, f_2, f_3, \ldots\ldots\ldots, f_n$, it means observation x_1 occurs f_1 times, observation x_2 occurs f_2 times and so on. Mean is denoted by $\bar{x}$.

There are three different ways to find the mean of a grouped data which are:

(a) Direct method.

(b) Assumed Mean method.

(c) Shortcut method (Step-deviation method).

Direct Method:

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{No. of observations}}$$

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots\ldots + f_n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

Assumed Mean Method:

$$\text{Mean } (\bar{x}) = a + \frac{\sum_{i=1}^{n} f_id_i}{\sum_{i=1}^{n} f_i}$$

where, a is any arbitrary value, chosen as assumed mean (somewhere in the middle of x_i), and $d_i = x_i - a$

(iii) Median of Grouped Data:

Condition I: When the data is discrete.

Step 1: Arrange data in ascending order.

Step 2: If the total frequency n is odd:

Then, $\left(\frac{n+1}{2}\right)^{th}$, observation is the median.

Step 3: If the total frequency n is even:

Then, mean of $\frac{n}{2}^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$, observations is the median.

Condition II: When the data is continuous and in the form of frequency distribution :

Then,

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c}{f}\right] \times h$$

Median class = The class whose cumulative frequency is greater than (nearest to) $\frac{n}{2}$.

where,

l = lower limit of median class

f = frequency of median class

h = class-size

n = number of observations

c = cumulative frequency of class preceding the median class.

(iv) Mode of Grouped Data: The class with maximum frequency is called the modal class.

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

where,

l = lower limit of the modal class

h = class-size

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

CHAPTER 14: Probability

- **Fundamentals:**
  1. **Experiment:** An operation which can produce some well defined outcomes.
  2. **Sample Space:** It is the total number of possible outcomes of a random experiment.
  3. **Event:** Any subset of sample space is called event.
  4. **Elementary Event:** Each outcome of any random experiment.
  5. **Sure Event (Certain event):** An event which always occurs whenever the random experiment is performed.
  6. **Impossible Event:** An event which never occurs whenever the random experiment is performed.
  7. **Favourable Event:** The cases which ensure the occurrence of an event.
  8. **Probability:** Probability $P(E)$ of an event E is defined as:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$$

or

$$P(E) = \frac{\text{Favourable Event}}{\text{Sample Space}}$$

  9. **Complement Events:** An event associated with a random experiment denoted by $P(\text{not } E)$ which happens only when E does not happen is called the complement of event E.

$$P(\bar{E}) \text{ or } P(\text{not } E) = 1 - P(E)$$

- **Tips:**
  1. Sum of the probabilities of all the elementary events of an experiment is 1.

$$P(E_1) + P(E_2) + P(E_3) + \ldots\ldots\ldots\ldots + P(E_n) = 1,$$

2. Probability of sure event is 1.
3. Probability of an impossible event is 0.
4. Probability of any event lies between 0 and 1 (including 0 and 1) i.e.,

$$0 \leq P(E) \leq 1.$$

5. 52 cards are divided into 4 suits of 13 cards is each. The suits are:

SPADE	♠	**HEARTS**	♥
DIAMONDS	♦	**CLUBS**	♣

6. Out of 52 cards 26 are red in colour and 26 are black.
7. In each suit there is an Ace, a King, a Queen, a Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
8. King, Queen and Jack are called face cards.